Quantifying Uncertainty

Robustness to Flaws, Geometric and Material Variability

Critical Exceedance Probabilities

Probability of crack tip force exceeding $\mathbf{G_{IC}}$ or $\mathbf{G_{IIC}}$ at 90 lb/in.

	G_{IC} [in-lbs/in^2]			G_{IIC} [in-lbs/in^2]		
Variability	**Mean**	**Weibull B-Value**	**Normal B-Value**	**Mean**	**Weibull B-Value**	**Normal B-Value**
Geometry	0	> 1 / 3	1 / 14,000	0	0	0
Geometry + Material	0	1 / 2	1 / 36	0	0	0

* MIL-HDBK-17 statistical procedures used.

Large variations in coupon measured fracture strengths will complicate test prediction.

Quantifying Uncertainty

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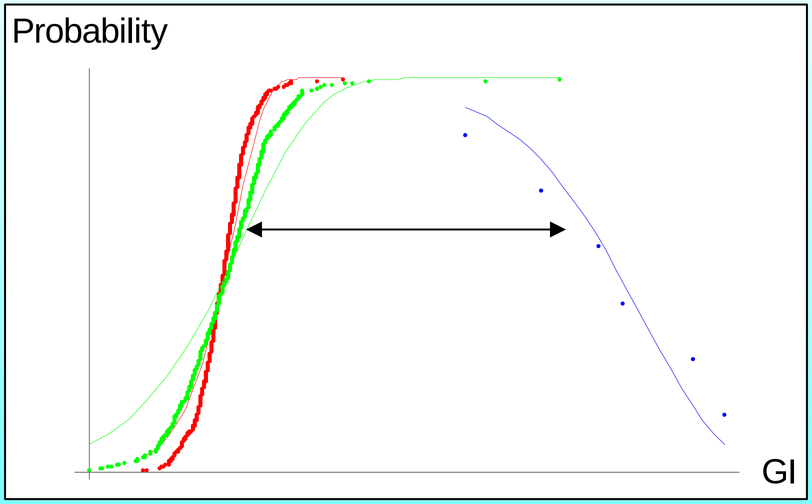
Load Exceedance Probability



For continuous distributions, the probability of failure is:

$$p_f = \int_0^\infty F_{G,sublam}(G_{max}) f_{G,exper}(G_{max}) dG_c$$

$F_{G,SUBLAM}$ is the CDF of expected SERRs for the HSP system

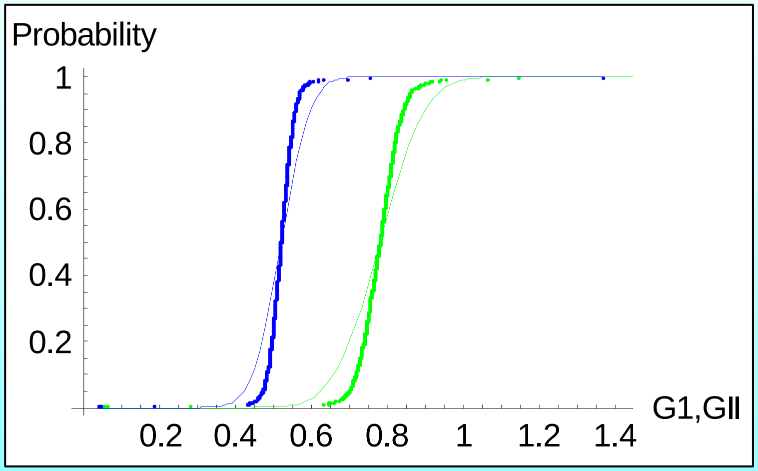
$f_{G,exper}$ is the PDF of the experimental data.

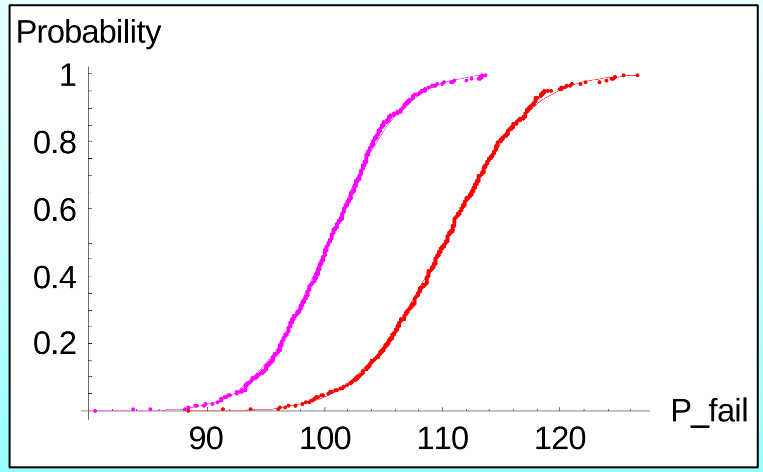
Quantifying Uncertainty

Robustness to Flaws, Geometric and Material Variability

Comparison to Distributions of G_{IC} and G_{IIC} (Left) and Pull-off Load (Right)

Pull off load = 80 lb/in





Interaction Criteria 1

$$\left(\frac{G_I}{G_{IC}}\right)^2 + \left(\frac{G_{II}}{G_{IIC}}\right)^2 = R1$$

Interaction Criteria 2

$$\frac{G_I}{G_{IC}} + \frac{G_{II}}{G_{IIC}} = R2$$

Quantifying Uncertainty

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Pull-off Failure Load Statistics

		Criteria 1	Criteria 2
Mean (lbs/in)		110	100
Standard Deviation		5.82	4.90
B-Values (lbs/in)	n = 6 (current number of experimental data)	77.5	72.6
	n = 10 (typical number of experimental data)	80.5	75.1
Weibull Distribution	n = 500 (simulation results)	99.8	91.6

B-value Prediction Strongly Depends on Confidence in Input Data

Data from Knowledge, Analysis, and Test

Using previous knowledge

- Test, analysis, and fabrication/service experience of similar materials and concepts, Lessons Learned
- Prone to Epistemic error and mistakes
  - 90% of the reports I've ever tried to directly use are missing at least one key piece of information that is required for my application.
  - Human memory can be faulty
- Divergence Risk – What constitutes similarity? How do you characterize or quantify differences?
  - We do this all the time (Engineering Judgment)
  - Example COV from similar systems
  - Mathematical or other structured approaches?
- Do we need new empirical knowledge?

Data from Knowledge – HSP Defects Example

Tooling Effect on Part Quality Producibility Heuristic Data (Excerpt)

Issue	Semi-Rigid Cocure Tooling	Cobond with Wet Hats
Thick/Thin Flanges	Flange thickness is a minor problem assuming semi-rigid section extends into bay between stiffeners. (<10% flange thickness error). Assume flange and skin under flange experience the same fiber volume change.	Flange edge thickness more variable. Flanges typically 15% thin due to tooling pressure. (Fiber volume change in flanges and skins under the flanges. Resin flowed out toward midbay and noodle area.)
Skin Waviness Beyond the Hat	Typically not a significant issue. A slight (<5%) thickness increase may be noted beyond stiffener flange.	Not an issue with precured skins
Shim Induced delamination at hat termination	Tooling is rigid enough to be pinned in place and prevent undercut by the shim. Some slight flange fiber movement over the shim is possible but can be trimmed back to the required shape	No shim required.
High/Low fiber volume at flange termination	Low fiber volume is common in net formed hats for ply pull back. Tooling approach does not significantly affect this.	Low fiber volume is common in net formed hats for ply pull back. Tooling approach does not significantly affect this.
End of hat thick or thin flanges	Limited intensifier droop near the end of the panel (5%)	Tooling flexibility will allow a roll-off or pinching at the hat termination. Expect the flanges to taper to 15% thin at tooling termination. If the hats are not net shape, this in not much of an issue.
Skin Waviness beyond the hat	The hat mandrel can create markoff beyond the end of the hat. Since this is typically a mating surface, shims are used to reduce this effect. Expect a 10% thickness decrease with shims.	Not an issue with precured skins
Tool mark-off	Tool mark off can be reduced by terminating the inner stiffening member before the flexible coatings.	Not an issue with precured skins

Data from Knowledge, Analysis, and Test
Data obtained by Analysis

- Relatively fast and inexpensive
- Easiest data type for dealing with most aleatory variations
- All analysis methods require input data obtained from test
  - True material scatter must be obtained from tests
  - Influence on failure load can be assessed by analysis
- Prone to Epistemic uncertainty
  - Is something missing in the Physics or Idealization?
  - More difficult as complexity of shape or loading increases
  - Surface Finish Example, Fillet Example
- Examples – Laminate Analysis, HSP pull-off.

Data from Knowledge, Analysis, and Test
Data Obtained from Small Tests

- Test Data is the current "Gold Standard"
  - Accurately Assesses Physics (of what is tested)
- More variation/error sources than generally recognized
  - Prone to excessive aleatory uncertainty
  - Specimen Prep and Test Setup variation not on the real aircraft
  - Example Uncertainty sources (FHC) lumped with "material scatter"
  - Example added test variation (OHC fixturing)
- Coupons and elements may not be representative of the actual structure unless excised from larger panels

Data from Analysis

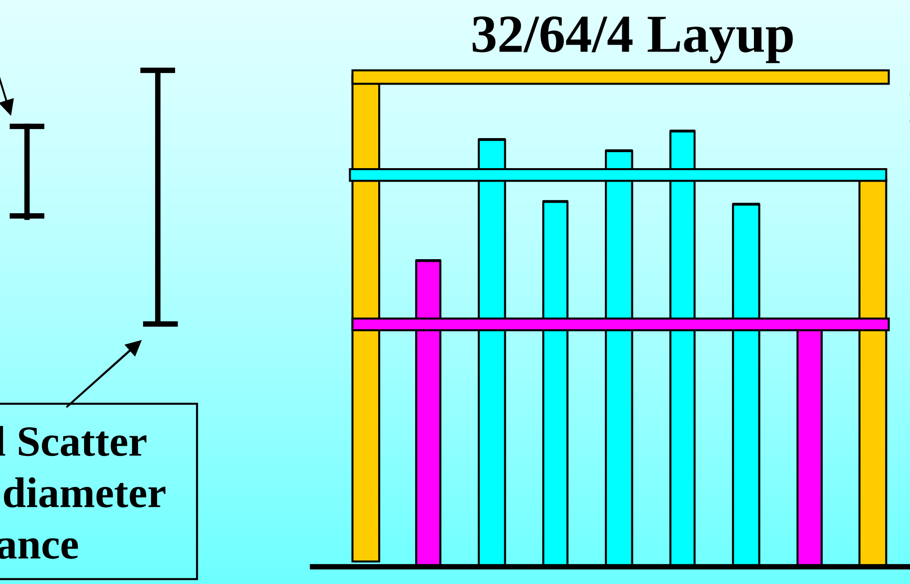
"Material Scatter" for FHC currently includes unknown hole fit

Filled-Hole Compression (FHC) Testing

Loose Hole - 0.005 in. Tolerance

Tight Hole – 0.002 in. Tolerance

Measured Scatter in Test Data



SIFT Prediction – Tight Hole, High Stiffness and Critical Failure Props

Test Data Average – Unknown Hole

SIFT Prediction – Loose Hole, Low Stiffness and Critical Failure Props

Potential Scatter Incl. Hole diameter tolerance

SIFT-Simulated Limits – Loose Hole

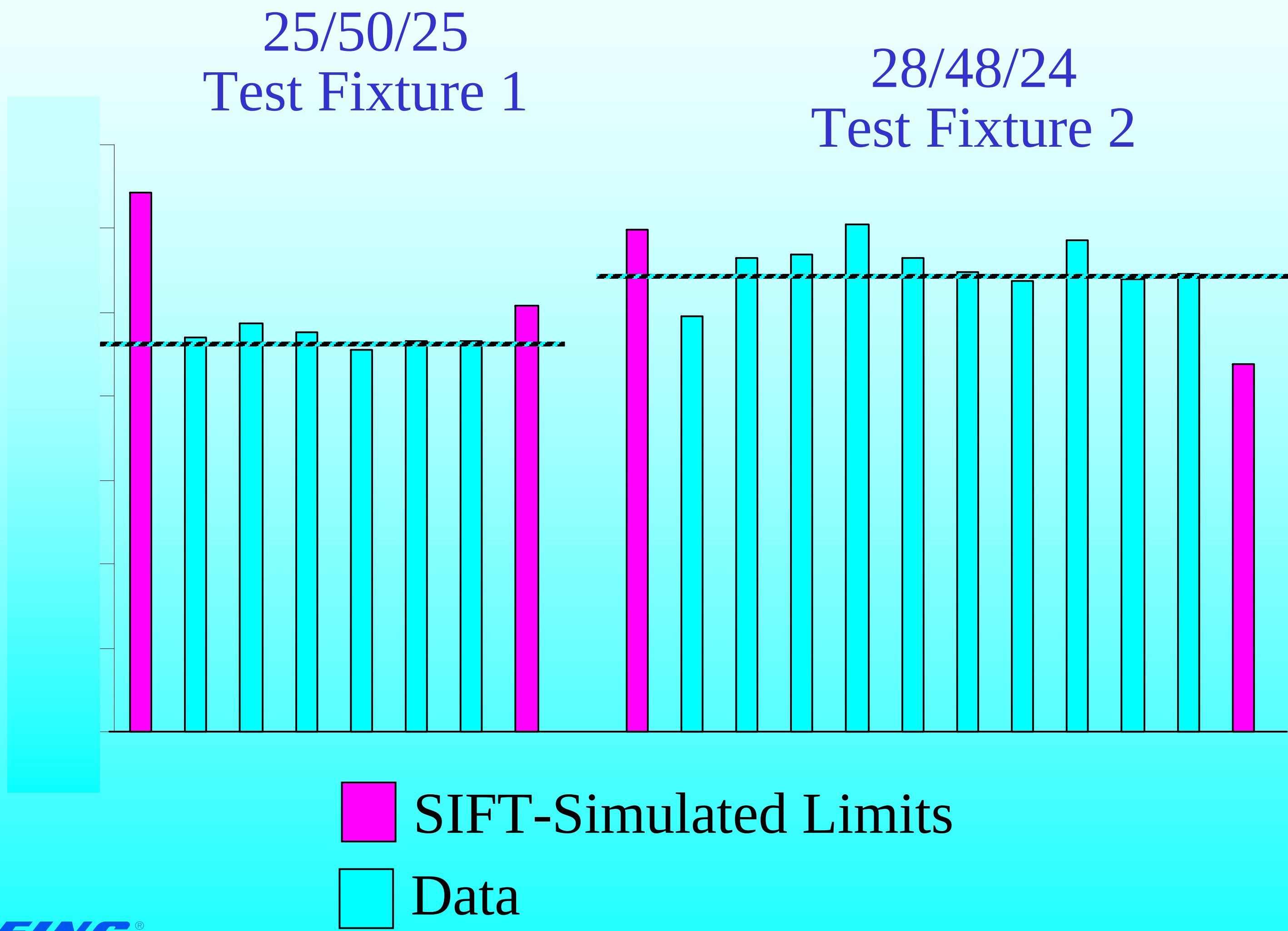
Test Data – Unknown Hole Clearance

SIFT-Simulated Limits – Tight hole

Data from Analysis

The Test Fixture and Method can significantly influence the results

Open-Hole Compression OHC Testing



Data from Knowledge, Analysis, and Test
Data Obtained from Large Tests

- Large-Scale Testing
  - Captures Scale-up effects (Manufacturing, Size)
  - Difficult to *Quantify* Aleatory Uncertainty
    - Few Replicates, Selected Environment/s, Single Critical Failure Mode
    - Relies on building blocks
    - Can often assess lower bound due to large number of repeating elements
  - Very Convincing (Looks "Real"), but still prone to "idealization errors"
    - Boundary Conditions, Loading, etc.
  - Concorde Durability Test Anecdote
- Great for validation…
  - Correct critical failure mode and location? Correct load distribution? Appropriate total correction for scatter? Nothing missed in physics?
- but expensive and insufficient if used alone

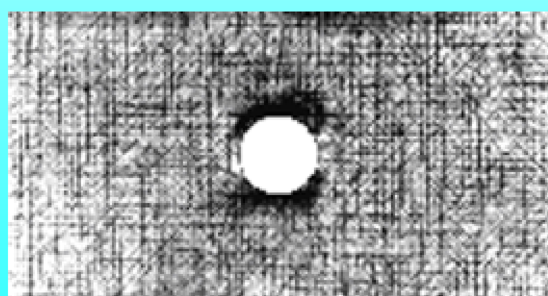
Data from Knowledge, Analysis, and Test Combined Data

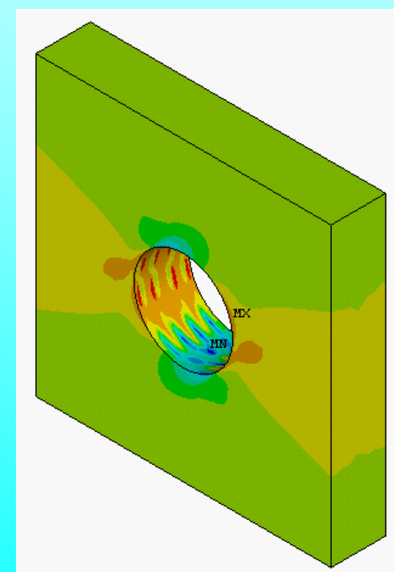
- Each data source (K,A,T) has its own unique characteristics and potential errors.
- We usually have data from all 3 sources.
  - How can we combine the data for maximum benefit/confidence?
- Corollary – Given our current K, what is the most effective combination of A and T to gain sufficient confidence in a material/design while minimizing time and $$?
  - Hierarchical Bayesian Approach?
  - Percentile Regression and Correlation to Analysis?
  - Allowables with Uncertainty?
  - Other?

Data from Knowledge, Analysis, and Test Combined Data

First Data Combination Problem

- **Composite Coupon Open-Hole Tension Test Data**
  - **Comparatively Simple**
  - **Several relatively-accurate analytical approaches**
- **Computational methods**
  - **The Strain Invariant Failure Theory (SIFT).**
  - **Point Stress Method.**





Data from Knowledge, Analysis, and Test Combined Data

- How can we combine the data for maximum benefit/confidence?
  - Hierarchical Bayesian Approach

$$\beta_k \approx N(\mu_{\beta_k}, \sigma^2_{\beta_k})$$

$$\delta \approx N(\mu_\delta, \sigma^2_\delta)$$

$$\eta_i = f_b(lam_i, \beta)$$

$$\eta_i = f_b(lam_i, \beta, \delta)$$

$$\sigma_b^2 \approx Gamma(3{,}2)$$

$$\mu_i \approx N(\eta_i, \sigma_b^2)$$

$$\mu_i \approx N(\eta_i, \kappa\sigma_b^2)$$

$$\kappa \approx Gamma(3{,}2)$$

$$\alpha_k \approx N(\mu_{\alpha_k}, \sigma^2_{\alpha_k}) \longrightarrow \theta_{ij} = f_s(x_{ij}, \mu_i, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$$

$$\sigma_S^2 \approx Gamma(3{,}1) \longrightarrow y_{ij} \approx N(\theta_{ij}, \sigma_s^2)$$

Hyperpriors are then defined for the unknown parameters in the priors

Data from Knowledge, Analysis, and Test Combined Data – Bayesian Approach

Advantages and Disadvantages of using Bayes Methodology

Pro's and con's for using Bayesian methods

While the primary motivation to use Bayesian reliability methods is typically a desire to save on test time and materials cost, there are other factors that should also be taken into account. The table below summarizes some of these "good news" and "bad news" considerations.

Bayesian Paradigm: Advantages and Disadvantages

Pro's	Con's
• Uses prior information - this "makes sense" • If the prior information is encouraging, less new testing may be needed to confirm a desired MTBF at a given confidence • Confidence intervals are really intervals for the (random) MTBF - sometimes called "credibility intervals"	• Prior information may not be accurate - generating misleading conclusions • Way of inputting prior information (choice of prior) may not be correct • Customers may not accept validity of prior data or engineering judgements • There is no one "correct way" of inputting prior information and different approaches can give different results • Results aren't objective and don't stand by themselves

NIST/SEMATECH e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/, August 2003.

Data from Knowledge, Analysis, and Test Combined Data

- How can we combine the data for maximum benefit/confidence?
  - Allowables with Uncertainty Bands

Goal: Predict, from computer runs simulating randomness, allowables.

Result: predicted allowables (which are inherently probabilistic statements) with uncertainty bands (not inherently probabilistic) generated by the Bayesian hierarchical method.

Advantage: Keeps the Bayesian analysis separated from the probability analysis.

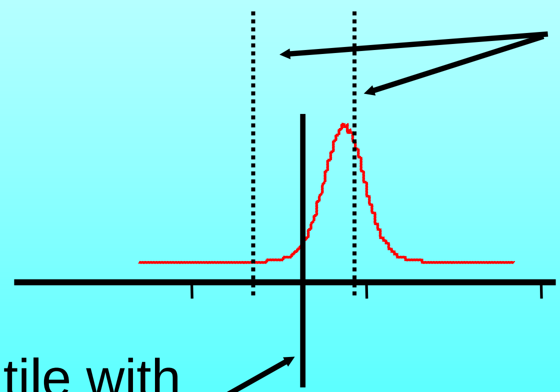
Data from Knowledge, Analysis, and Test Combined Data – Allowables with Uncertainty

•Data contain replicates => can estimate stress allowables (quantiles with confidence bands)

•RDCS allows simulation of physical data with sources of randomness including batch effects (aleatory or random uncertainty) => can simulate allowables.

•Combined data: allowables with uncertainty bands

Bayesian uncertainty band on allowable



Allowable estimate = quantile with confidence band. This is the “aleatory” content

Aleatory and Bayesian are kept separate

Data from Knowledge, Analysis, and Test Combined Data

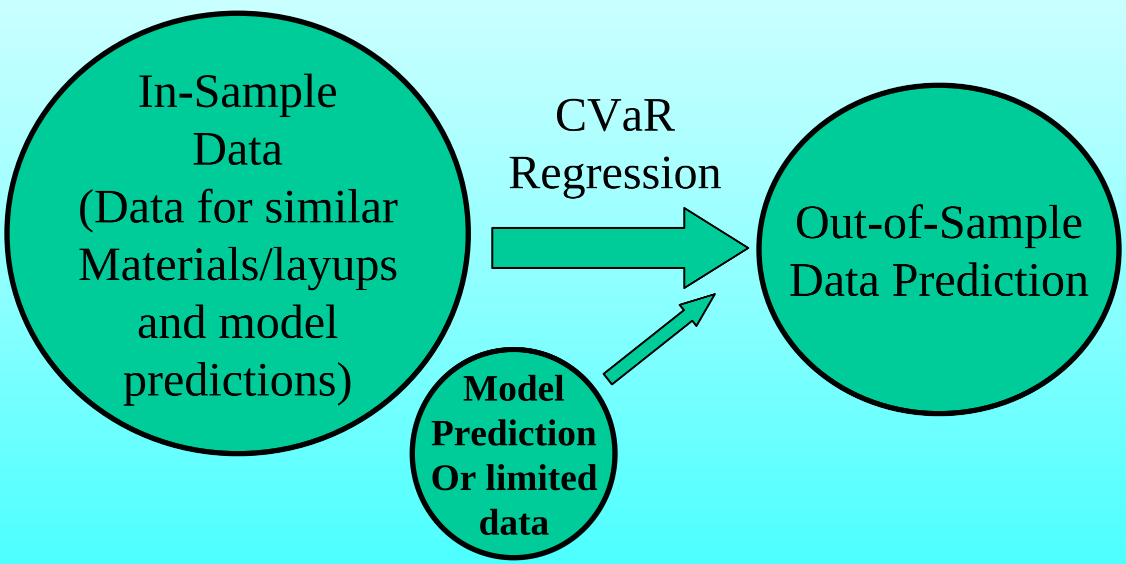
- How can we combine the data for maximum benefit/confidence?
  - Percentile Regression and Correlation to Analysis
  - Model prediction calibration in the stochastic domain with pooled test data using weighting factors
    - More accurate calibration of scatter, lower 10^{th} percentile etc.
  - As a second step demonstrate calibrated model prediction capability for a different condition not used to calibrate the model
  - Use of results from models of two different fidelity
    - Extensive use of Approximate Point Stress Model
    - SIFT model
      - Plan to enhance the current LL and UL prediction to Probabilistic

Data from Knowledge, Analysis, and Test Combined Data

- Studied available approached for combining experimental and modeling data:
  - Bayes models
  - Factor models
    - Parametric (normal / log-normal distribution) regression
    - Non-parametric: percentile and CVaR regression
- Effort was concentrated on Factor CVaR regression models as the most promising approach
- Made calculations for the dataset of experimental and modeling data (open hole coupon data) using CVaR regression

Data from Knowledge, Analysis, and Test Combined Data

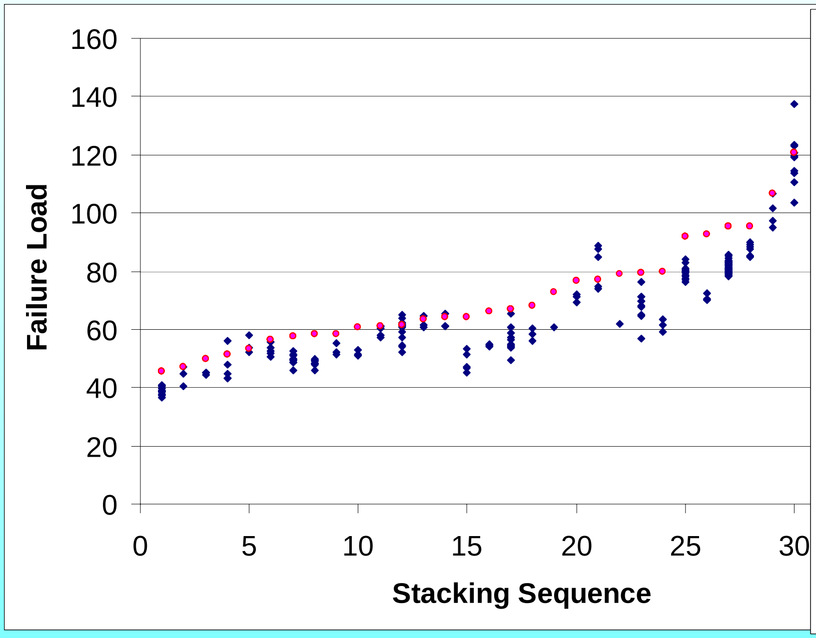
- Preliminary calculation results showed that CVaR regression approach provides reasonable results. Three situations were considered:
  - **- predictions of percentiles using SIFT outputs (see the next slide)**
  - **- predictions of percentiles by pooling experimental data**
  - **- predictions of percentiles by combining model and experimental data**



- Formal approach has been identified for calculation of B-basis in the framework of CVaR regression. However, it has not jet been implemented.

Data from Knowledge, Analysis, and Test Combined Data

CVaR REGRESSION: SIFT MODEL PREDICTIONS / DATA

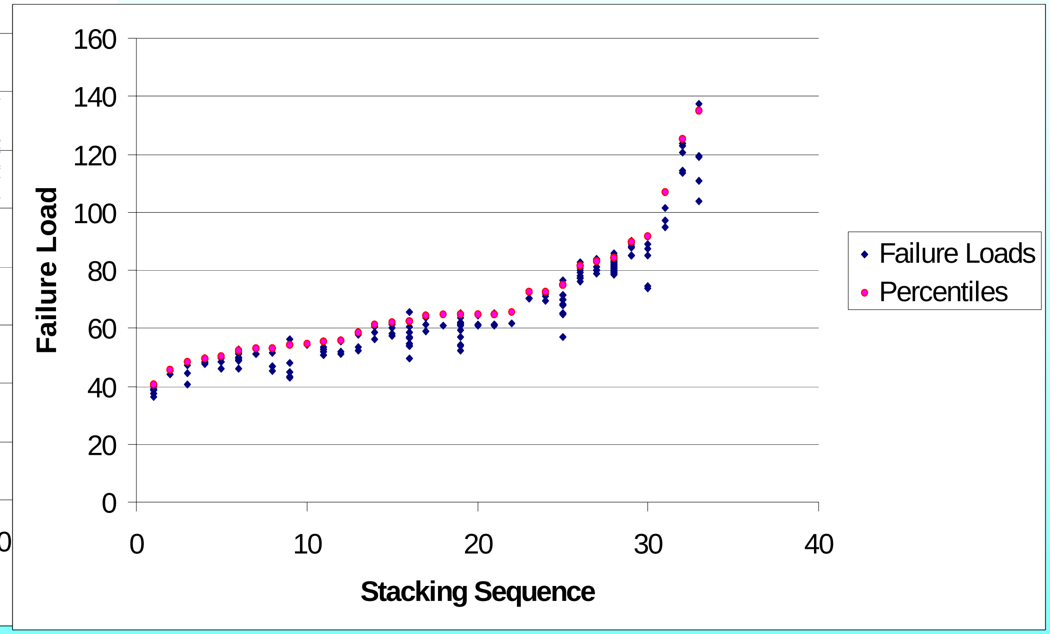


Model Only

$$y^{i}_{90\%} = 8.12 + 1.063\,(\mu^{i} - \overline{\mu}) + 0.334\,(\sigma^{i} - \overline{\sigma})\ ,$$

$$\overline{\mu}\ = 68.7\ ,\qquad \overline{\sigma} = 4.39$$

$$CVaR^{\Delta}_{\alpha} = 11.96$$



Model + One Test Data Point

$$y^{i}_{90\%} = 3.9 + 0.12\,(\mu^{i} - \overline{\mu}) - 0.69\,(\sigma^{i} - \overline{\sigma}) + 0.92\,(\mu^{i} - \overline{\mu})\ ,$$

$$\overline{\mu}\ = 73.63\ ,\qquad \overline{\sigma} = 4.41,\qquad \overline{\mu} = 69.01$$

$$CVaR^{\Delta}_{\alpha} = 7.89$$

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